

## STABILITY OF ANNULAR FLOW IN HORIZONTAL TUBES

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(Received 26 October 1980; in revised form 26 August 1981)

**Abstract**—A model for transition between stratified and annular two-phase flows in horizontal tubes has been developed, based upon a momentum balance and energy transfer between the two phases. Without specifying the mechanisms involved in the transition, a criterion has been put forth which requires that the mechanical energy change experienced by the gas be greater than the work done by the interfacial forces in bringing about the transition. In this sense, the proposed criterion is “global” and concerns itself only with the states of the two-phase flow before and after the transition. The criterion predicts a region of stability for annular flow which is in reasonable agreement with the Baker plot as well as other transition maps. It has been demonstrated that the transition depends upon the void-fractions in stratified and annular flows and thus helps to explain the wide discrepancies in the observed data from several sources.

### INTRODUCTION

In horizontal tube two-phase flows, there exist several flow patterns. Alves classified the observed patterns into the following categories: stratified, wavy, slug, plug, bubbly, froth flow and annular or annular mist flow which finally degenerates into dispersed flow. In the design of pipes which carry liquid–gas mixtures (as in the petroleum industry) and in heat exchangers where phase changes occur, it is necessary to know the pressure drop and the heat transfer, both of which are strongly dependent on the flow pattern. Hoogendoorn (1959) for example, has demonstrated that for essentially similar flow conditions, slug or wave flow may be attained in air–water mixtures, resulting in changes in pressure drop by a factor of 2. The calculations of Bell *et al.* (1970) show that the use of stratified flow correlations could result in condensation heat transfer estimates that are several orders of magnitude different from those by using annular flow correlations. Thus, in order to obtain an accurate estimate of the pressure drop and heat transfer in a pipe, it is necessary to know the actual flow pattern at any point in the pipe.

Several attempts have been made to predict the flow pattern for specified liquid and gas mass flow rates or superficial velocities. Kosterin (1949), Alves (1954), Baker (1954), Krasiakova (1954) and Hoogendoorn (1959) were among the first few to propose maps which could be utilized for the prediction of flow patterns in adiabatic flow. These maps show various regions into which the flow patterns fall, when two selected correlation parameters are plotted against each other. Several other investigators have also proposed empirical maps which are usually modifications of earlier maps based on more extensive data, e.g. Govier & Omer (1962), Scott (1963), Govier & Aziz (1972) and Mandhane *et al.* (1974). In most of these maps (see for example, figures 1 or 2), lines of demarcation among the various flow regimes are shown. In practice, however, there are no sharp distinguishing boundaries and the type of flow changes gradually from one to another as the parameters change.

Empirical maps to predict transitions among the flow patterns in condensation are also available. Traviss & Rohsenow (1971) observed the flow regimes in horizontal two-phase flow with condensation. Soliman & Azer (1974) formulated a flow pattern map based upon observations in tubes ranging from 0.476 to 1.59 cm.

Among the flow patterns that can exist, annular/annular–mist and stratified flow are the most important and cover a quality range between them from 1 to 90% or more. In flows with phase

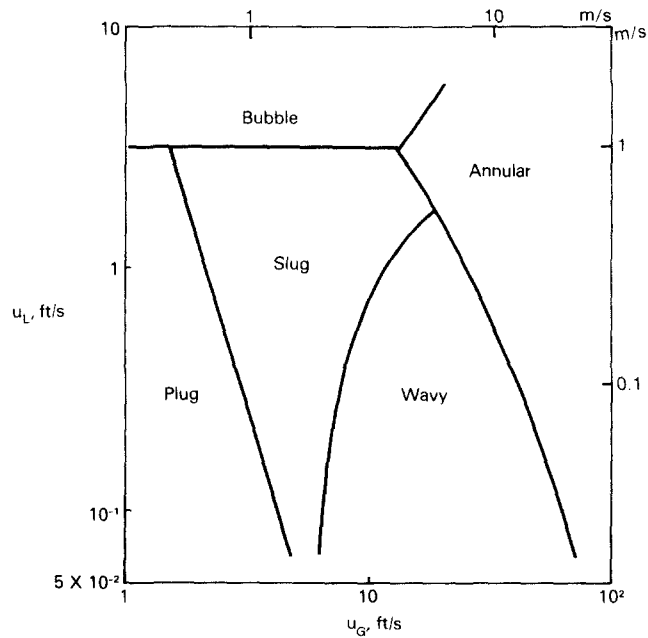


Figure 1. Alves flow map.

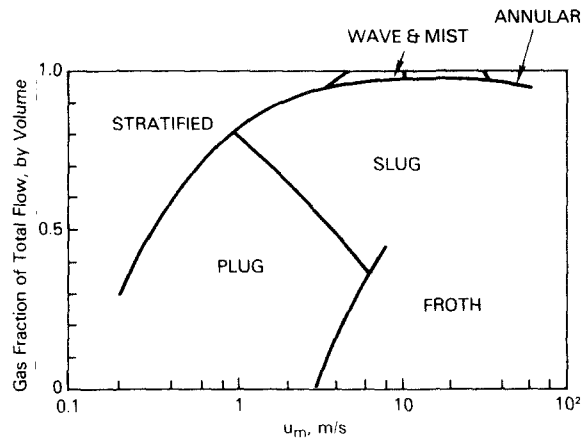


Figure 2. Hoogendoorn map.

changes, as much as 95–98% of the heat transfer can occur in these regimes. So, a considerable amount of attention has been focused on the annular stratified transition. Jaster & Kosky (1976) have defined an empirical force balance parameter which represents the ratio of gravitational and shear forces. This factor is expected to define the existence of annular or stratified flow depending upon its magnitude. Wallis (1962) has also defined a force balance parameter for vertical flows. According to Palen *et al.* (1979), the same parameter can be used to determine annular–stratified transitions in horizontal tubes as well.

In spite of all the effort expended so far, there has been no general agreement among the investigators about even the independent variables to be used in the flow pattern maps. Alves (1954) used the superficial liquid and gas velocities, whereas Baker (1954) used the mass flux ratio for the abscissa and the gas mass flux for the ordinate. While Govier & Omer (1962) used the liquid and gas mass fluxes as the coordinates, Govier & Aziz (1972), Mandhane *et al.* (1974), and Dukler (1978) recommend the use of superficial liquid and gas velocities. Several correction

factors to take account of changes in tube diameters and liquid as well as gas properties have been proposed, based mostly on observations. So far, none of the flow maps has dimensionless coordinates. Another major difficulty is that data from several sources do not necessarily agree with one another. There are strong indications that changing diameters, centrifugal forces, the existence of bends, heat transfer, etc. alter the flow patterns significantly (see for example, Weisman *et al.* 1979,† Hughes 1953 and Isbin *et al.* 1959). Further, as stated earlier, a mere variation in inlet conditions has produced slug as well as wave patterns at the same liquid and gas mass flow rates. It was for these reasons that Scott (1963) proposed a modified Baker plot showing areas of flow transition in place of lines. Even with this map, Scott recommends that further allowances be made for tube diameters 2.5 cm and lower, since the indicated areas tend to change rapidly with decreasing tube sizes. Mandhane (1974) has made allowances for property changes by defining a parameter,  $F$ , which is itself a function of the liquid-gas properties such as densities, viscosities and surface tension.

Taitel & Dukler (1975) have proposed a mechanistic model for flow transitions among several flow patterns. They use the Kelvin-Helmholtz condition for the stability of a wavelet to determine the criteria for transition from stratified to slug flows. They assert further that annular flow results if the stratified liquid level before transition is below the pipe center line, while slug flow results otherwise. The results obtained therefrom and the use of empirical factors are in agreement with the Mandhane (1974) flow map, though not with the Baker (1954) and Scott (1963) maps everywhere. Taitel & Dukler (1977) have extended the model to unsteady flow with changing gas flow rates, to predict slug frequency. Niu & Dukler (1978) have also presented an intermittent heat transfer calculation based on these ideas.

The mechanism that maintains a full annular film in a horizontal tube has been the subject of conjecture as well as investigation by several authors. Jacowitz (1962) argues that the only stable situation is one that is dynamic, where a balance exists among the several forces (lift, surface drag, pressure, wall drag, wave formation effect, etc.), with a small but continuous downward drainage of liquid in the film. The film is expected to become thinner and thinner as the two-phase flow proceeds along the tube. Butterworth (1971) and Butterworth & Pulling (1972) have shown that waves play an important role in maintaining the film at the top of the tube. Butterworth concludes that the liquid annulus can be maintained due to a continuous replenishment of the film at the top by one of two possible mechanisms. The first involves a secondary circulation in the film against gravity caused by differences in liquid film thickness between the bottom and the top (Darling & McManus 1968). The second is the action of waves which spread the liquid from the high flow to the low flow regions (Butterworth, 1969a and 1969b). A third mechanism which is favored by some authors (e.g. Fisher & Pearce 1978) is that the liquid drains continuously under gravity and is replenished purely by entrainment and redeposition of the liquid droplets due to the high velocity gas flow. Several observations (Krsiakova 1952, Wicks & Dukler 1960) indicate either negligible entrainment or decreasing entrainment with increased liquid flux. It is difficult to see how entrainment can be the sole mechanism to maintain the film when annular flows have been reported with only 5% entrainment or less. What is possible is that several of these mechanisms aid one another in maintaining a stable film, with entrainment being an unimportant contributor at the start of annular flow.

Since the exact mechanisms that cause a change from stratified to annular flow are very complicated, little understood and hard to analyze, an attempt has been made here to predict the transition requirements based upon "global" considerations. It is postulated that annular flow arises when the energy exchange between the phases is sufficient to raise the liquid energy to that in annular flow. The force of interaction between phases is pictured as the agency responsible for the energy exchange between the phases. The transition criterion is based on

†The author is grateful to the reviewer who brought this reference to his notice.

the requirement that the gas should undergo a total energy change that is greater than the work done by the interactive forces between the gas and the liquid. Thus, the proposed criterion is completely different from that of Taitel & Dukler (1975), which postulates the instability of a wavelet at the interface in stratified flow as responsible for the transition. The present postulate leads to a transition condition depends upon the specified annular and stratified void-fractions and slip, as well as on the nondimensional groups, the Froude number, Weber number and Reynolds number. Calculations demonstrate that annular flow can occur only if the liquid Froude number is close to unity. This implies that for stability, the liquid kinetic energy in annular flow be greater than its potential energy at the center of the tube. A comparison with the Scott version of the Baker plot shows very good agreement for air–water flows in 10 cm (4 in.) dia. tubes. For 5 cm (2 in.) and 2.5 cm (1 in.) dia. tubes, the criterion specifies larger stable annular regions of flow than specified by the map. Moreover, the stable region is shown to depend upon the inlet and exit void-fractions utilized as the input and thus helps to explain the discrepancies among the results of various investigators as well as the necessity for establishing a fully stable flow before making measurements.

#### ANALYSIS

The attempt here is to model transition between the annular and stratified flows. This transition is the most important in the study of two-phase heat exchangers since almost 95% of the total energy exchange occurs in annular and stratified flow regimes. Hence, consideration will be limited to this transition, though it is expected that the analysis will predict the transition boundary between annular and the rest of the patterns in its neighborhood, with sufficient accuracy.

Let a two-phase liquid gas mixture enter a horizontal pipe in annular flow, with a specified void-fraction. If the flow induced is unstable, it will change to stratified conditions as shown in figure 3, over a transition zone. Similarly, if it is started in stratified flow, it will change into annular flow, provided annular flow is stable under the impressed conditions. Thus the direction of flow indicated in the figure is not of much consequence in the analysis. We simply consider the flow on one side of the transition zone (indicated by subscript 1) as annular and on the other side (indicated by subscript 2) as stratified.

Let  $u_L$  and  $u_G$  be the respective average liquid and gas velocities, flowing through a tube of radius  $R$ . The intention is to make mass and momentum balances over the transition region, assuming steady, incompressible flows coming into and going out of the control volume. In addition, heat transfer is neglected so that the present analysis applies only to adiabatic conditions. Entrainment and redeposition of droplets which usually occurs in annular flow, is neglected in this first analysis. If sufficient information on droplets is available, they can be included and modified results obtained.

In annular flow, the gas core has always been observed to be eccentric with respect to the pipe, resulting in a thicker liquid film at the bottom of the tube than at the top. It is assumed that the gas core is also circular with radius  $R$ , and its center displaced from that of the pipe as shown in figure 4. For given values of the eccentricity,  $h$ , and radii  $R$ ,  $R_c$ , it is readily demonstrated from geometry that the liquid layer thickness,  $\delta$ , is related to the angle  $\theta$  at the

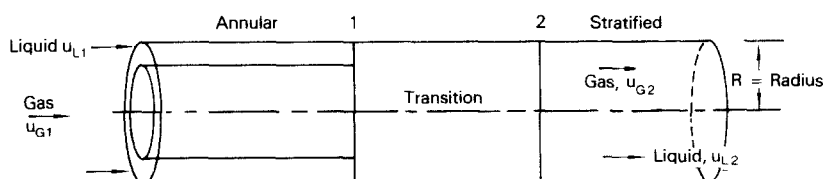


Figure 3. Annular-stratified transition.



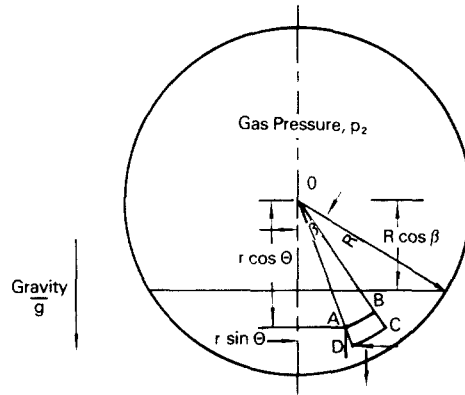


Figure 5. Stratified flow.

The total axial force on the liquid due to pressure is:

$$\begin{aligned}
 F_{L2} &= 2 \int_0^\pi \int_{R \cos \beta / \cos \theta}^R [p_2 + \rho_L g (r \cos \theta - R \cos \beta)] r dr d\theta \\
 &= -2 \int_0^\beta \left[ \left( p_2 \frac{R^2}{2} - \frac{\rho_L R^3 \cos \beta}{2} \right) \left( \frac{\cos^2 \beta}{\cos^2 \theta} - 1 \right) + \rho_L g \frac{\cos \theta R^3}{3} \left( \frac{\cos^3 \beta}{\cos^3 \theta} - 1 \right) \right] d\theta. \\
 &= (p_2 R^2 - \rho_L g R^3 \cos \beta) (\beta - \sin 2\beta/2) + 2\rho_L g R^3 \sin^3 \beta/3. \quad [5]
 \end{aligned}$$

Note that in annular flow the void-fraction is related to  $R_i$  by the relation:

$$\alpha_1 = R_i^2 / R^2, \quad [6a]$$

while in stratified flow, the void fraction is related to the angle,  $\beta$ , by the relation

$$\alpha_2 = 1 - (\beta - \sin 2\beta/2) / \pi. \quad [6b]$$

By using [4], [5], [6a, b] and noting that the axial forces due to pressure on the gas sections 1 and 2 are, respectively:

$$F_{G1} = \pi R_i^2 p_1 \text{ and } F_{G2} = \pi R^2 \alpha_2 p_2, \quad [7]$$

we can write the momentum equation in the axial direction in the following form:

$$F_{G1} + F_{L1} + (\text{Mom})_1 = F_{G2} + F_{L2} + (\text{Mom})_2 + F_s$$

i.e.

$$\begin{aligned}
 &\pi R^2 \left[ p_1 \alpha_1 + \left( p_1 - \frac{\sigma}{R_i} + \frac{\rho_L g h}{2} \right) (1 - \alpha_1) + \rho_G u_{G1}^2 \alpha_1 + \rho_L u_{L1}^2 (1 - \alpha_1) \right] \\
 &= \pi R^2 [p_2 \alpha_2 + (p_2 - \rho_L g R \cos \beta) (1 - \alpha_2) + 2\rho_L g R \sin^3 \beta / (3\pi) + \rho_G u_{G2}^2 \alpha_2 + \rho_L u_{L2}^2 (1 - \alpha_2)] + F_s. \quad [8]
 \end{aligned}$$

Here,  $F_s$ , is the shear force due to wall effects and other dissipative forces in the transition region and  $\rho_G$  is the gas density. Experimental observations in the previously cited literature as well as those conducted by the present author in a 1.9 cm tube indicate that the transition region

is itself short, compared with the tube length. It is likely that frictional forces which are usually proportional to tube length, are small in this regime and may be neglected as compared with the rest of the quantities in the above equation.

We write the continuity equations as follows:

$$u_{G1}\alpha_1 = u_{G2}\alpha_2 \quad (\text{gas volume flow}) \quad [9a]$$

$$u_{L1}(1 - \alpha_1) = u_{L2}(1 - \alpha_2) \quad (\text{liquid volume flow}). \quad [9b]$$

Equations [9a] and [9b] are based on the assumption that the gas temperature does not change appreciably during transition and that the densities of both phases may be considered as constant in view of the small pressure change,  $p_1 - p_2$ .

With these relations and assumptions [8] can be simplified to obtain an expression for the pressure change during transition:

$$\frac{\Delta p_R}{\rho_L u_{L1}^2} = (\alpha_2 - \alpha_1) \left( \frac{1 - \alpha_1}{1 - \alpha_2} - R_d S_1^2 \frac{\alpha_1}{\alpha_2} \right) + \frac{1}{\pi Fr^2} \left[ \sin \beta \left( 1 - \frac{\sin^2 \beta}{3} \right) - \beta \cos \beta - \frac{\pi h (1 - \alpha_1)}{2R} \right] + \frac{(1 - \alpha_1)}{We \sqrt{\alpha_1}} \quad [9]$$

In this equation, the quantity  $\Delta p_R$  represents the pressure difference,  $p_1 - p_2$ , assuming frictional effects during transition to be negligible while  $R_d = \rho_G / \rho_L$ . The quantities,  $Fr$  and  $We$ , are, respectively, the Froude Number and the Weber number, defined as follows:

$$Fr = u_{L1} / (gR)^{1/2}; \quad We = \rho_L u_{L1}^2 R / \sigma. \quad [10]$$

The Froude and Weber numbers based on liquid superficial velocities and tube diameter are related to the above quantities by the respective equations:

$$Fr^* = (1 - \alpha_1) Fr / \sqrt{2} = u_{Ls} / (gD)^{1/2} \quad [11a]$$

$$We^* = 2(1 - \alpha_1)^2 We = \rho_L u_{Ls}^2 D / \sigma. \quad [11b]$$

In these equations,  $u_{Ls}$ , is the liquid superficial velocity and  $D$ , the tube diameter. For a given liquid-gas combination (air-water for example), the Weber number is specified once the tube radius and Froude number are fixed.

For specified values of  $\alpha_1$ , the annular void-fraction, and  $\alpha_2$ , the stratified void-fraction, Froude and Weber numbers, the velocity ratio  $S_1 = u_{G1} / u_{L1}$ , and annulus eccentricity,  $h$ , it is now possible to compute the pressure change,  $\Delta p_R$ , that will occur in the event of transition. Since the interest here is in specifying the line of stability of annular flow, computations have been carried out assuming that the eccentricity is the maximum permissible for the given value of  $\alpha_1$ , i.e. one where the liquid-gas interface is just tangential to the tube inner surface. Then:

$$h = R - R_i = R(1 - \sqrt{\alpha_1}). \quad [12]$$

Experimentally, it should be possible to produce almost any specified void-fraction and slip-ratio, at either the inlet to the annular or the stratified section. Indeed, Hasson & Nir (1970), through a suitable combination of a nozzle and fluid flow arrangement produced various inlet annular conditions to study the stability of such flows. Hence, one can choose any arbitrary values of  $\alpha_1$  and  $\alpha_2$  ( $0 \leq \alpha_1 \leq 1$ ,  $0 \leq \alpha_2 \leq 1$ ), slip, etc. and require [9] to provide the pressure difference for that particular case. For calculational purposes, what has been done is to pick the void-fractions and slip for annular flow by the use of the Pai (1953) velocity profiles and the

Wallis (1970) interfacial friction factors, as shown briefly in Appendix A. Similarly, for stratified flow, the curve-fits provided by Kadambi (1981) and exhibited below have been utilized:

(a) Turbulent–turbulent flow:

$$\alpha_2 = \frac{1 + 1.4133 X_{tt}}{1 + 2.5315 X_{tt} + 0.4403 X_{tt}^2} \quad (\text{Re}_L \leq 2000; \text{Re}_G > 2000) \quad [13a]$$

(b) Laminar–turbulent flow:

$$\alpha_2 = \frac{1 + 1.9566 X_{Lt}}{1 + 3.0927 X_{Lt} + 0.6946 X_{Lt}^2} \quad (\text{Re}_G \geq 2000; 1200 \leq \text{Re}_L < 2000). \quad [13b]$$

For superficial liquid Reynolds numbers lower than 1200 in the laminar–turbulent range, the following equations are used:

$$1 - \alpha_2 = (1 - \alpha'_2)[1 + 0.161 \times 10^{-3} (\text{Re}_L - 1200)] \quad (500 \leq \text{Re}_L \leq 1200) \quad [13c]$$

$$1 - \alpha_2 = (1 - \alpha'_2)[1 + 0.246 \times 10^{-3} (\text{Re}_L - 1200)] \quad (120 \leq \text{Re}_L \leq 500) \quad [13d]$$

$$1 - \alpha_2 = (1 - \alpha'_2)[1 + 0.31 \times 10^{-3} (\text{Re}_L - 1200)] \quad (25 \leq \text{Re}_L \leq 120) \quad [13e]$$

In the above equations,  $\alpha'_2$  is the void-fraction calculated from [13b].

(c) Laminar–laminar flow:

$$\alpha_2 = \frac{1 + 0.6684 X_{LL}}{1 + 1.2424 X_{LL} + 0.201 X_{LL}^2} \quad [13f]$$

In [13a]–[13f], the quantities  $X_{tt}$ ,  $X_{Lt}$  and  $X_{LL}$  are the Lockhart–Martinelli parameters defined by the following equations:

$$X_{tt} = [(1 - x)/x]^{0.9} R_d^{0.5} / M^{0.1} \quad [14a]$$

$$X_{Lt} = 18.6501 (\text{Re}_L)^{0.1} (R_d \text{Re}_L)^{0.5} (1 - x)/x \quad [14b]$$

and

$$X_{LL} = [(1 - x)/x]^{1/2} (R_d M)^{1/2}. \quad [14c]$$

#### THE TRANSITION CRITERION

Having determined the pressure change during the expected transition, it is necessary to propose a criterion which will specify which of the two states, annular or stratified, will exist. A survey of the literature reveals that one of two propositions is usually used in similar situations. These are: (i) the minimum entropy production principle, and (ii) the principle of minimum energy.

For the present situation, the use of the minimum entropy production principle would lead to the assertion that the stable configuration is that which has the lower dissipation and, therefore, the lower pressure drop (between annular and stratified flows) for the same liquid and gas mass fluxes. The minimum energy principle, on the other hand, leads to the assertion that the stable state is that which has less mechanical energy between the two possible ones. Moreover, both the criteria ignore the transition region where the liquid–gas interaction leading



to the change in flow regimes occurs. It is, therefore, concluded that they are not applicable to the present situation.

Wallis & Dobson (1973) have correlated the transition from stratified to slug flow for rectangular ducts in terms of the dimensionless gas superficial velocity,  $j_G^*$ , and the void-fraction  $\alpha$ , by the relation:

$$j_G^* = 0.5\alpha^{3/2}. \quad [15]$$

The  $3/2$  power of  $\alpha$  in the above equation has been obtained from the use of the Kelvin-Helmholtz classical theory of instability (Milne-Thompson, 1968), while the coefficient, 0.5, is empirical. The Taitel-Dukler (1975) analysis also uses a result obtained from the Kelvin-Helmholtz principle, except for a coefficient chosen in a form which meets the limiting conditions as  $\alpha$  tends to zero or unity in stratified flow. Taitel and Dukler make the further assertion that annular flow occurs due to the instability if the equilibrium liquid level in stratified flow is lower than the pipe centerline. The basis for such an assertion, however, is not clear.

None of these criteria can be utilized here directly. As such, we shall examine the transition region and the interaction as well as the energy transfers between the liquid and the gas more closely. Since a combined momentum equation for the liquid and gas has already been written, we write an equation for the gas alone, to determine the force of interaction between the gas and the liquid during the transition. By considering a differential volume element of the gas, a force balance yields:

$$\dot{m}_G du_G = -A\alpha dp + dF_i. \quad [16]$$

Here,  $dF_i$  is the force of interaction between the gas and the liquid element which may ultimately cause the transition. Once again shear and dissipative forces have been neglected as done for the overall momentum balance.

Since the void-fraction,  $\alpha$ , varies during the transition, it will not be possible to integrate this equation exactly. An approximate result can, however, be obtained by assuming that the effect on  $F_i$  of the variation in  $\alpha$  is not large for flows close to transition and replace  $\alpha$  by its mean value at its end states, i.e. by  $\alpha_m = 0.5(\alpha_1 + \alpha_2)$ . Then, [16] can be integrated directly and leads to the result:

$$\begin{aligned} F_i &\approx -A[\alpha_m \Delta p_R + \rho_G u_{G1} \alpha_1 (u_{G1} - u_{G2})] \\ &= -A\alpha_1 \left[ \frac{\alpha_m}{\alpha_1} \Delta p_R + \rho_G u_{G1}^2 \frac{(\alpha_2 - \alpha_1)}{\alpha_2} \right]. \end{aligned} \quad [17]$$

Here,  $\Delta p_R$  is the pressure change in a transition obtained by neglecting all dissipative effects.

The force,  $F_i$ , is responsible for energy transfer between the liquid and the gas. If flow should become annular starting from a stratified condition, it is necessary that the gas should provide the difference in liquid energies between the two regimes through the medium of this interactive force which acts at the interface. The rate of work done by the gas on the liquid is, therefore:

$$W = F_i u_i = -\dot{m}_L \frac{\alpha_1 u_i u_{L1}}{(1 - \alpha_1)} \left[ \frac{\alpha_m}{\alpha_1} \frac{\Delta p_R}{\rho_L u_{L1}^2} + \frac{R_d S_1^2 (\alpha_2 - \alpha_1)}{\alpha_2} \right] \quad [18]$$

where  $u_i$  is the average interfacial velocity in the transition regime.

Since the gas does work on the liquid during the transition it suffers an energy loss between

stratified and annular flows by an amount equal to:

$$\begin{aligned}\Delta E_G &= \dot{m}_G[(p_2/\rho_G + u_{G2}^2/2) - (p_1/\rho_G + u_{G1}^2/2)] \\ &= -\rho_G u_{G1} A \alpha_1 \left[ \frac{\Delta p_R}{\rho_G} + \frac{u_{G1}^2 - u_{G2}^2}{2} \right] \\ &= -\dot{m}_L \frac{1}{1 - \alpha_1} u_{L1} u_{G1} \left[ \frac{\Delta p_R}{\rho_L u_{L1}^2} + \frac{R_d S_1^2}{2} \left( \frac{\alpha_2^2 - \alpha_1^2}{\alpha_2^2} \right) \right].\end{aligned}\quad [19]$$

It is now postulated that the proposed flow transition occurs provided the gas energy difference between the two regimes,  $\Delta E_G$ , is at least as large as the energy transfer at the interface, i.e. stable annular flow can occur only if:

$$\Delta E_G \geq -F_i u_i. \quad [20]$$

On substituting for  $\Delta E_G$  and  $F_i u_i$  from [19] and [18] and simplifying, we obtain:

$$-u_{G1} \left[ \frac{\Delta p_R}{\rho_L u_{L1}^2} + \frac{R_d S_1^2 (\alpha_2^2 - \alpha_1^2)}{2 \alpha_2^2} \right] \geq u_i \left[ \frac{\alpha_m}{\alpha_1} \frac{\Delta p_R}{\rho_L u_{L1}^2} + R_d S_1^2 \frac{(\alpha_2 - \alpha_1)}{\alpha_2} \right]$$

or:

$$\frac{\Delta p_R}{\rho_L u_{L1}^2} \left( 1 + \frac{\alpha_m u_i}{\alpha_1 u_{G1}} \right) \leq -\frac{R_d S_1^2 (\alpha_2 - \alpha_1)}{\alpha_2} \left( \frac{\alpha_m}{\alpha_2} + \frac{u_i}{u_{G1}} \right). \quad [21]$$

The quantity,  $u_i$ , the average interfacial velocity during the transition, is not easily determined. It is probably safe to assume that  $u_i$  is of the same order of magnitude as the interfacial velocities in annular and stratified flows. It is known through observations (Armand 1946; Bergelin & Gazley 1949; Agrawal *et al.* 1973) and theoretical calculations (Kadambi 1980a, 1980b) that the interfacial velocities in both annular and stratified flows are quite small being often less than 4–5% of  $u_{G1}$ . If we, therefore, neglect terms containing  $u_i/u_{G1}$  compared with unity, the transition criterion reduces to the form:

$$\frac{\Delta p_R}{\rho_L u_{L1}^2} \leq \frac{R_d S_1^2 (\alpha_1^2 - \alpha_2^2)}{2 \alpha_2^2}. \quad [22]$$

It is thus asserted that transition to annular flow occurs and a stable flow results if the inequality in [21] (or the approximate form in [22]) is satisfied. If not, stratified flow will be stable, since the gas does not have enough energy to bring about the transition to annular flow.

Since energy exchange between the gas and the liquid is the postulated mechanism for the transition, it is clear that the gas energy in annular flow is less than that in stratified flow under the same mass flow rates and other conditions, i.e.  $\Delta E_G$  must be positive. From [19] it follows that:

$$\frac{R_d S_1^2 (\alpha_1^2 - \alpha_2^2)}{2 \alpha_2^2} - \frac{\Delta p_R}{\rho_L u_{L1}^2} \geq 0.$$

This requirement is automatically satisfied if the transition criterion in [22] is met. Also,

$$\begin{aligned}-F_i u_i &= \frac{\dot{m}_L \alpha_1 u_i u_{L1}}{(1 - \alpha_1)} \left[ \frac{\alpha_m}{\alpha_1} \frac{\Delta p_R}{\rho_L u_{L1}^2} + \frac{R_d S_1^2 (\alpha_2 - \alpha_1)}{\alpha_2} \right] \\ &\leq \frac{\dot{m}_L \alpha_1 u_i u_{L1}}{1 - \alpha_1} \left[ \frac{\alpha_m^2 R_d S_1^2 (\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2^2} + \frac{R_d S_1^2 (\alpha_2 - \alpha_1)}{\alpha_2} \right] \\ &\leq \frac{\dot{m}_L \alpha_1 u_i u_{L1}}{1 - \alpha_1} \frac{(\alpha_1 - \alpha_2)^3}{4 \alpha_1 \alpha_2^2} R_d S_1^2.\end{aligned}\quad [23]$$

Since  $F_i u_i$  is the energy supplied to the interface by the gas, this quantity should be positive as well. The product  $u_i u_{L1}$  is positive and  $\alpha \leq 1$ . Therefore, all the terms on the r.h.s. of [23] will be positive if:

$$\alpha_1 > \alpha_2 . \quad [24]$$

Hence, a necessary condition for transition is that the *annular void-fraction be greater than the stratified void-fraction*. This condition ensures that the annular gas velocity is lower than that in stratified flow. The same conclusion may be reached by looking at the gas-liquid energy transfers as well. Since the liquid is lifted up in going from the stratified to the annular state, it is reasonable to suppose that the total liquid energy in the annular state is higher than that in the stratified state. The total mechanical energy of the flow can at best remain constant during the transition, so that an increment in liquid energy can occur only at the expense of the gas kinetic energy due to the interaction postulated earlier. Hence, the gas velocity in annular flow must be lower than that in stratified flow.

#### RESULTS AND DISCUSSION

By using equilibrium values of void-fractions in annular and stratified flows, the pressure change required for transition is obtained from [9]. Subsequently, the transition criterion specified by the inequality in [21] has been used to predict the stability of annular flow. Most of the calculations are for air-water mixtures in 2.5, 5 and 10 cm tubes, though a few calculations have been carried out for gas-oil systems as well, with the properties specified by Baker (1954). Some of the calculated results as well as a comparison with data available in the literature, are presented in the following.

Figure 6 shows the input values of the void-fraction difference,  $\alpha_1 - \alpha_2$ , plotted against the annular void-fraction,  $\alpha_1$ , for 2.5 cm dia. tubes. Annular void-fractions were obtained by using the calculations specified in Appendix A, while the stratified void-fractions were obtained from [13a]-[13f], both at the same liquid and gas mass flow rates. The Froude number specified by

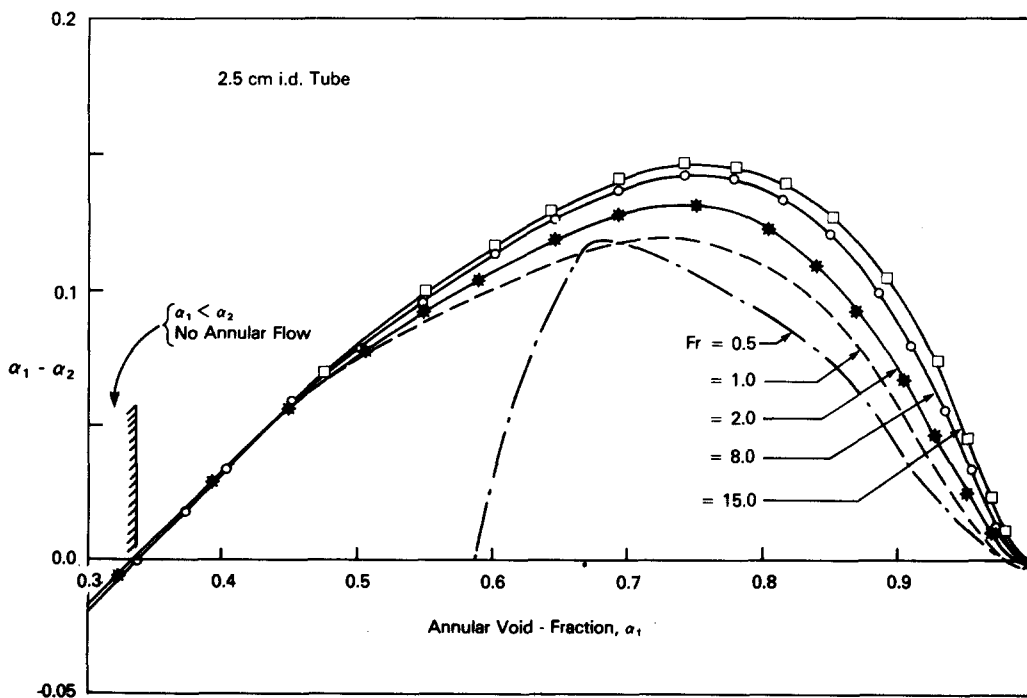


Figure 6. Void-fractions in annular and stratified flows.

[10] has been used as a parameter in the plot. It is seen that the stratified void-fraction is usually smaller than the annular void-fraction, the difference between the two increasing with increasing Froude numbers. In addition, for every Froude number, there exist some conditions where  $\alpha_2$  is greater than  $\alpha_1$ . These observations apply to tubes of other diameters as well.

By using these void-fractions, predictions of stable annular flow regions have been made and the results are shown in figure 7. Here, the superficial liquid and gas Reynolds numbers have been plotted respectively as the abscissa and the ordinate, with  $FrWe^{3/2}$  as a parameter. Regions of stable annular flow are seen to be dependent on tube diameter and are shown by shading the inside of the curves for 2.5, 5 and 10.1 cm tubes. In agreement with the observations of Scott (1963), the smaller diameter tubes tend to go into annular flow more easily than large diameter tubes. One expects intuitively that this trend is correct, since the potential energy change between stratified and annular flows is much less in a small tube than in a large tube.

Figure 7 also depicts that there exists a certain Froude number below which no transition can occur. This value turns out to be approx. 0.75 for the three diameters considered. Since the Froude number as defined here involves only the average liquid velocity and the tube radius, it implies that in annular flow the average liquid kinetic energy should at least be equal to its potential energy at the tube center. This is an observation that is amenable to experimental verification.

Since energy exchange between the liquid and the gas is the postulated mechanism that causes annular flow, a large gas kinetic energy and a sufficiently large interfacial area are necessary for the transition. If, therefore, holding the gas velocity constant, the liquid mass flow rate and thus its superficial velocity are reduced, the interfacial area of contact between the liquid and the gas gradually decreases in stratified flow. Thus, the chances of transition to annular flow are continuously reduced with decreasing liquid flow rates. This is what is implied by the Baker plot, figure 8, which shows larger and larger requirements of gas mass flux,  $G_G$ , with decreasing values of  $G_L/G_G$ . In figure 7, this effect manifests itself as a limiting value of  $Re_L$  for a given tube diameter, below which no annular flow can occur.

Figure 8 is the Scott (1963) version of the Baker plot, on which the predictions of the present theory for 2.5, 5.1 and 10.2 cm tubes are shown for comparison. For the 10.2 cm tube, the predicted transition line agrees closely with that of the Baker maps, except near the slug flow boundary. The calculations for the smaller sized tubes indicate larger stable regions of

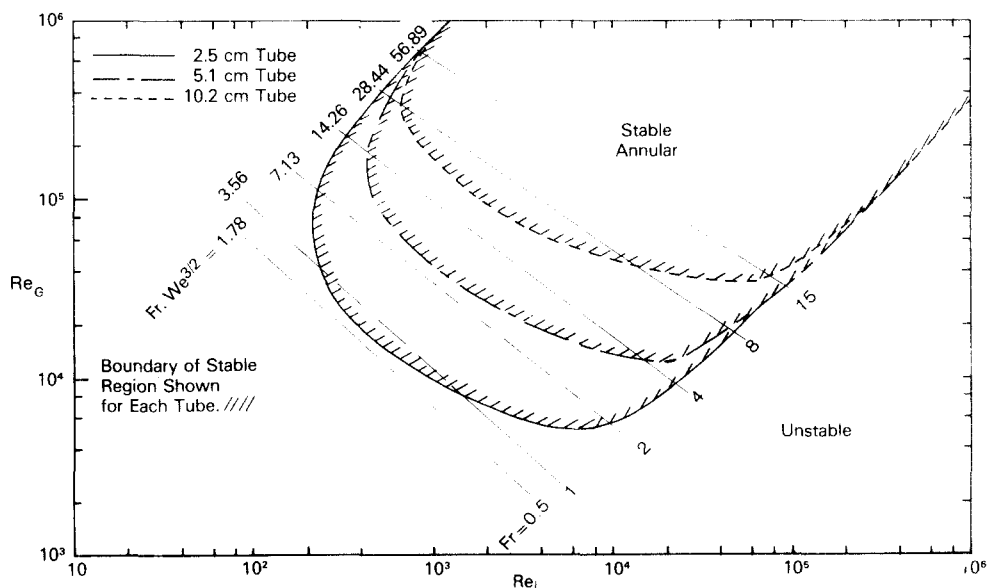


Figure 7. Predicted annular-stratified boundaries for various tube diameters.

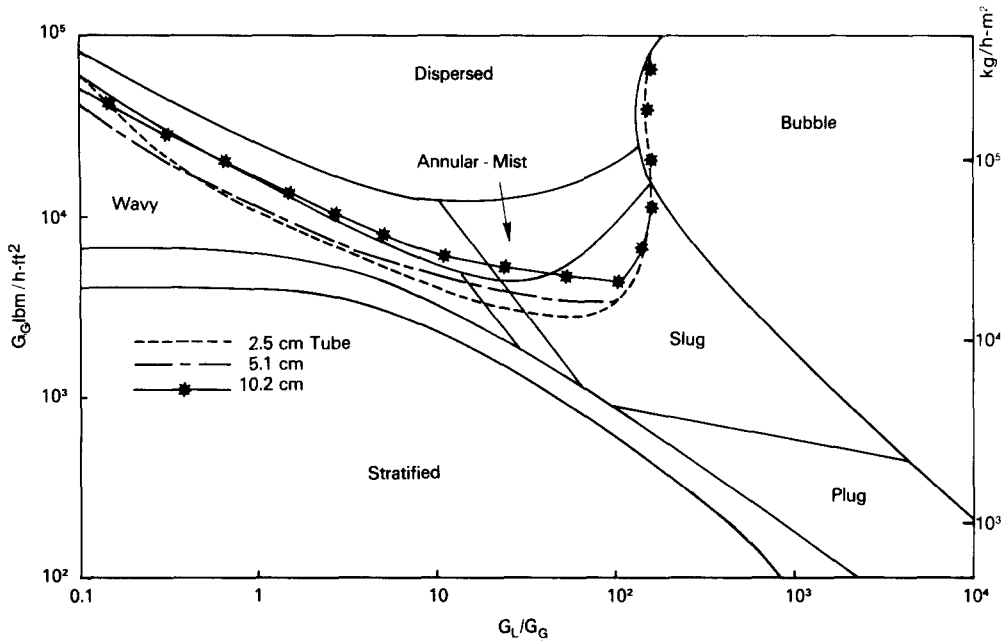


Figure 8. Comparison of predictions with Scott map.

annular flow than the map. Moreover, this predicted diameter effect is in general agreement with the experimental data of Weisman *et al.* (1979). The limited calculations carried out for oil-air two-phase flow provide similar results, though they have not been shown on the map.

The disagreement between the present theory and the map in the slug region arises partly due to the void-fractions used in the annular regime. As seen from the Appendix A, the void-fractions were calculated by using the Pai velocity in turbulent flow along with the Wallis equation (1970):

$$f_i/f = 151 - 150\sqrt{\alpha} \tag{25}$$

where,  $f_i$  is the actual interfacial friction factor, while  $f$  is the friction factor that would exist if the same two-phase flow existed with a smooth annular interface. This equation is valid only for large values of  $\alpha$ . The values of  $f_i$  obtained for large liquid flow rates (small  $\alpha$ ) are inaccurate and thus, the predicted region of stability is not very accurate in the slug regime. Moreover, slug flow does not approximate stratified flow which has been used as the model in this theory. For both these reasons, some discrepancy between the theory and observations ought to be expected at the boundary of this regime.

A comparison among some of the flow maps as well as the Taitel-Dukler and present theories as applicable to the annular transition region, is shown in figure 9. Here, the superficial velocities are the coordinates. The lack of agreement among the maps is evident. The Mandhane *et al.* (1974) region of stable annular flow is the line 1-1 and the Baker region is line 2-2. Line 3-3 is the Taitel-Dukler (1975) theory while line 4-4 is the present prediction. For low liquid superficial velocities, both the Mandhane line and the Taitel-Dukler line indicate a region of stable annular flow, as long as the gas velocity is in excess of 70-80 ft/s (23-26 m/s). This result is in disagreement with the Baker plot and the results of Weisman *et al.* (1979). It also disagrees with the results of some simple experiments that the author has run. In addition, according to both the Baker plot and the present theory, higher and higher gas velocities are needed to achieve annular conditions as the liquid velocity is reduced. At high liquid velocities, the present theory seems to overpredict the region of stability, as already noted. If data regarding

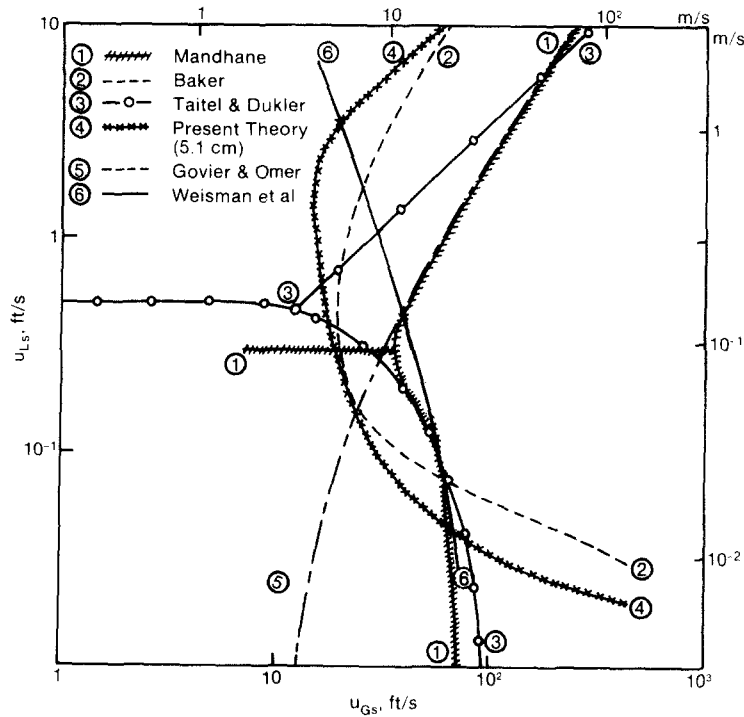


Figure 9. Comparison of predictions with Mandhane plot.

void-fractions in annular flow at relatively high liquid fluxes become available, or if a better interfacial friction factor correlation is obtained, the results of the present prediction can be improved.

In order to see how sensitive the predicted stability is to the input variables, two sets of "trial" calculations were attempted. In the first one, the values of  $\alpha_1$ ,  $S_1$ , etc. were all chosen as the solutions to the annular flow problem with laminar liquid, laminar gas conditions, irrespective of the Reynolds numbers. Similarly, the stratified flow void-fraction was obtained by using [13f], which is valid only in laminar flow. The stability criterion then predicts a totally stratified flow, independently of the Froude and Weber numbers. A second set of "trial" calculations involved changing the values of  $\alpha_1$ ,  $\alpha_2$ , etc. slightly from those used in the graph, figure 6. These indicate that annular flow stability usually improves with increasing values of  $\alpha_1 - \alpha_2$ , and diminishes with decreasing values when the other parameters are all fixed. Thus, the type of flow finally obtained in a given experiment for the same flow rates of liquid and gas, can depend strongly on the conditions at the test section inlet and exit. This may be the major reason for the large amount of disagreement among the several investigators exhibited in figure 9. In accordance with the recommendations of Hewitt (1976), the transition maps should, therefore, be used purely as guides and no reliance should be placed upon them to provide the exact regions of stability.

Unfortunately, no simple method which can be used to extend the results to other gas-liquid combinations and diameters exists. Efforts are presently under way to calculate similar results for other cases as well. Curve fits that permit easy calculations to predict stability will then be obtained.

It is to be noted here that the stability criterion developed is independent of the procedure used to calculate the annular and stratified void-fractions. The present author's procedures (Kadambi 1980, 1981) have been used to predict  $\alpha_1$  and  $\alpha_2$ , simply because they have been found to be quite accurate in their agreement with data from several sources, especially for air-water flows. There is no requirement or necessity to use these procedures to determine the

equilibrium void-fractions in annular and stratified flows. Other procedures that predict these void-fractions accurately can be used equally well.

Since the stability criterion provided by [21] involves  $u_i$ , the interfacial transition velocity whose value is not known accurately, some calculations have also been carried out to determine the sensitiveness of the results to changes in assumed values of  $u_i$ . As long as  $u_i$  is less than about 8% of  $u_{G1}$  in magnitude, the predicted region of stability is almost independent of its magnitude. For all practical purposes, therefore, [22] is as good as [21] for transition calculations.

All the present calculations consider annular flow as complete when a liquid film of zero thickness forms at the top of the tube, with an eccentricity as specified by [12]. Experimental observations are, however, subject to parallax errors and it is impossible to tell when the annular film has just formed completely. Usually, a liquid film of finite thickness exists at the tube top when one recognizes the flow as annular. This results in data that tend to underestimate the range of stable annular flow, especially when transitions to the slug and bubble flow patterns are under investigation. Therefore, the present theory can be expected to overpredict the region of annular flow in comparison with observations. In addition, since frictional and dissipative effects have been neglected, one expects a further slight overestimation of the annular regime. If corrections are incorporated to obtain the proper value of  $\Delta p$  including dissipative effects in transition, one can expect better agreement between the predictions of the present model and observations. Considering the several simplifications used, the agreement between experimental observations (as specified by the flow maps) and the present theory may be considered as quite satisfactory, except near the annular-slug boundary.

It must once again be emphasized that the present theory concerns itself only with the conditions required for the existence of stable flow regimes, without considering the details of the transition process. Therefore, it does not propose a mechanism for the transition or for the causes of the instability of a given flow pattern. In this respect, it is quite different from the Taitel-Dukler (1975) model, which proposes the Kelvin-Helmholtz stability criterion as the requirement for stratified flow. The present theory indicates strongly that a transition to annular flow occurs due to an energy exchange between the liquid and the gas at the interface, without specifying the mechanism for the transfer. This postulate has been seen to be successful in predicting transition in several situations as already shown and thus provides a new insight to the problem as a whole. It also suggests strongly that entrainment and redeposition have no influence in the film formation, since the equations used here exhibit stability with no entrainment at all. This supports the contention of Butterworth (1971) that wave-spread or secondary flow maintains the annular film at the top of the tube against gravity. It is possible that both these mechanisms complement each other and are also aided by entrainment at a later stage.

#### CONCLUSIONS

1. Based on momentum balance and energy exchange considerations a criterion for the existence of stable annular flow has been established. This criterion predicts a line of transition between annular flow and the rest of the flow patterns which is different for different tube sizes. The smaller the tube size, the larger the range of stable annular flow.

2. The predicted range of stability is in reasonable agreement with the Baker plot. At very low liquid flow rates, it predicts results which are in better agreement with experimental observations than the Mandhane plot.

3. According to the present theory, stable annular flow can occur only when the liquid kinetic energy is equal to or larger than the liquid potential energy at the tube center. If it is smaller, the flow cannot be annular.

4. Two-phase flow stability seems to be strongly affected by the specified conditions at the

entry to the test-section, even when the mass flow rates and fluid properties are fixed. This may explain the large degree of disagreement among the experimental data in specifying the ranges of stability of the various flow patterns.

*Acknowledgements*—The author is grateful to Mr. Tom Martel (presently with Bendix Corporation, NJ), who spent several hours in discussing the problem and whose intuitive logic provided badly needed inputs at several stages of work. Dr. Ralph Wood provided technical discussions during the work and helped clear up several "cobwebs" in the author's mind. Mr. Karl Hardcastel and Mr. Peter Morgan assisted in the experimental observations as well as picture taking. The author is also grateful to the General Electric Company for permitting external publication of this paper.

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## APPENDIX A

## DETERMINATION OF ANNULAR FLOW VOID-FRACTIONS

A very brief account of the method used to predict annular flow void-fractions is provided below. The details of the procedure used may be obtained from Kadambi (1980).

Considering an annular flow in a horizontal tube, the liquid velocity is calculated by using the Pai (1953) profile:

$$u_L = u_{mL} \left[ 1 + \frac{s_L - n}{n - 1} \eta^2 + \frac{1 - s_L}{n - 1} \eta^{2n} \right] \quad [A1]$$

Here,  $\eta = r/R$ , is the non-dimensionalized radial coordinate, while  $s_L$  and  $n$  are given by the equations:

$$\begin{aligned} s_L &= 2.4172 \times 10^{-12} (\text{Re}_L)^{3.51} & (2040 \leq \text{Re}_L \leq 2800) \\ &= 0.585 + 0.0013172 (\text{Re}_L)^{0.833} & (\text{Re}_L > 2800) \\ n &= -0.617 + 0.008211 (\text{Re}_L)^{0.786}, & (n \text{ integer}) \end{aligned}$$

Here,  $\text{Re}_L$  is the Reynolds number that would exist for the liquid flowing through the pipe with the maximum velocity  $u_m$ .

Similarly, we use for the gas velocity profile the equation:

$$u_g - u_i = U_m \left[ 1 + \frac{s_G - m}{m - 1} \left( \frac{\eta}{\eta_i} \right)^2 + \frac{1 - s_G}{m - 1} \left( \frac{\eta}{\eta_i} \right)^{2m} \right]. \quad [A2]$$

The values of  $s_G$  and  $m$  are specified similar to those of  $s_L$  and  $n$ . The quantity  $u_i$  is the interfacial velocity obtained by putting  $\eta = \eta_i$  in [A1].

The interfacial shear is obtained by using the Wallis (1970) equation as specified by [25] of the main text. Entrainment has been taken into account by using Hutchinson & Whalley's (1973) correlation,† provided in the form of a graph.

The velocity profiles have been integrated to determine the mass flow rates of the liquid and the gas, and thus the mixture quality. Wall shear and pressure drop have been determined from the equations of motion, after the matching requirements at the interface are completed.

Numerical calculations based on these equations provide results in very good agreement with several sources, in particular for air-water flows.

†Hutchinson, P. & P. B. Whalley 1973 A possible characterization of entrainment in annular flow. *Chem. Engng. Sci.* **28**, 974-975.